Quadratic Simultaneous Equations

Exam Style Questions

1. Solve the following simultaneous equations

$$y = x^2 + 8x + 17$$
$$y = 13x + 31$$

$x^2 + 8x + 17 = 13x + 31$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = 7, \quad x = -2$

when $x = 7$

$y = 13(7) + 31$

$= 91 + 31$

$= 122$

when $x = -2$

$y = 13(-2) + 31$

$= -26 + 31$

$= 5$

$= (7, 122)$

or $(-2, 5)$

$x =$ 7 or -2

$y =$ 122 or 5

(5 marks)

2. Solve the following simultaneous equations

$$x^2 + y^2 = 10$$
$$y = x + 2$$

$x^2 + (x+2)^2 = 10$

$x^2 + x^2 + 4x + 4 = 10$

$2x^2 + 4x + 4 = 10$

$2x^2 + 4x - 6 = 0$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$

or $x = 1$

when $x = -3$

$y = -3 + 2 = -1$

when $x = 1$

$y = 1 + 2 = 3$

$(-3, -1)$

or $(1, 3)$

$x =$ -3 or 1

$y =$ -1 or 3

(5 marks)

3. Solve the following simultaneous equations

$$x^2 + 3y^2 = 28$$
$$y = x + 2$$

$x^2 + 3(x+2)^2 = 28$

$x^2 + 3(x^2 + 4x + 4) = 28$

$x^2 + 3x^2 + 12x + 12 = 28$

$4x^2 + 12x - 16 = 0$

$x^2 + 3x - 4 = 0$

$(x+4)(x-1) = 0$

$x = -4$ or $x = 1$

when $x = -4$
$y = -4 + 2$
$= -2$

when $x = 1$
$y = 1 + 2 = 3$

$= (-4, -2)$ or $(1, 3)$

$x =$ -4 or 1

$y =$ -2 or 3

(5 marks)

4. Solve the following simultaneous equations

$$x^2 + y^2 = 65$$
$$x - y + 7 = 0$$

$x = y - 7$

$(y-7)^2 + y^2 = 65$

$y^2 - 14y + 49 + y^2 = 65$

$2y^2 - 14y - 16 = 0$

$y^2 - 7y - 8 = 0$

$(y-8)(y+1) = 0$

$y = 8$ or $y = -1$

when $y = 8$: $x = 8 - 7 = 1$

when $y = -1$: $x = -1 - 7 = -8$

$= (1, 8)$ or $(-8, -1)$

$x =$ 1 or -8

$y =$ 8 or -1

(5 marks)

5. Solve the following simultaneous equations

$$x^2 + y^2 = 1$$
$$x - y = 1$$

$x = y + 1$

$(y+1)^2 + y^2 = 1$

$y^2 + 2y + 1 + y^2 = 1$

$2y^2 + 2y = 0$

$2y(y+1) = 0$

$y = 0$ or $y = -1$

when $y = 0$: $x = 0 + 1 = 1$

when $y = -1$: $x = -1 + 1 = 0$

$= (1, 0)$ or $(0, -1)$

$x =$ 1 or 0

$y =$ 0 or -1

(5 marks)

6. Solve the following simultaneous equations

$$x^2 + 2y^2 = 20$$
$$x = y + 3$$

Give your answers correct to 2 decimal places.

$(y+3)^2 + 2y^2 = 20$

$y^2 + 6y + 9 + 2y^2 = 20$

$3y^2 + 6y - 11 = 0$

$y = \dfrac{-6 \pm \sqrt{(6)^2 - 4(3)(-11)}}{2(3)}$

$= \dfrac{-6 \pm \sqrt{36 + 132}}{6}$

$= \dfrac{-6 \pm \sqrt{168}}{6}$

$y = \dfrac{-6 + \sqrt{168}}{6} = 1.16$

$y = \dfrac{-6 - \sqrt{168}}{6} = -3.16$

when $y = 1.16$

$x = 1.16 + 3$

$= 4.16$

when $y = -3.16$

$x = -3.16 + 3$

$= -0.16$

$= (4.16, 1.16)$

or $(-0.16, -3.16)$

$x =$ 4.16 or −0.16

$y =$ 1.16 or −3.16

(5 marks)

7. Solve the following simultaneous equations

$$x^2 + y^2 = 6$$
$$y = 3x - 4$$

Give your answers correct to 2 decimal places.

$x^2 + (3x-4)^2 = 6$

$x^2 + 9x^2 - 24x + 12 = 6$

$10x^2 - 24x + 6 = 0$

$5x^2 - 12x + 3 = 0$

$x = \frac{12 \pm \sqrt{(-12)^2 - 4(5)(3)}}{2(5)}$

$= \frac{12 \pm \sqrt{144 - 60}}{10}$

$= \frac{12 \pm \sqrt{84}}{10}$

$x = \frac{12 + \sqrt{84}}{10} = 2.12$

$x = \frac{12 - \sqrt{84}}{10} = 0.28$

when $x = 2.12$

$y = 3(2.12) - 4$

$= 2.36$

when $x = 0.28$

$y = 3(0.28) - 4$

$= -3.16$

$= (2.12, 2.36)$

or $(0.28, -3.16)$

$x =$ 2.12 or 0.28

$y =$ 2.36 or -3.16

(5 marks)

8. Solve the following simultaneous equations

$$x^2 - y^2 = 5.25$$
$$2x + 3y = 4$$

$2x = 4 - 3y$

$x = \frac{4-3y}{2}$

$\left(\frac{4-3y}{2}\right)^2 - y^2 = 5.25$

$\frac{16 - 24y + 9y^2}{4} - y^2 = 5.25$

$16 - 24y + 9y^2 - 4y^2 = 21$

$5y^2 - 24y - 5 = 0$

$(5y + 1)(y - 5) = 0$

$y = -\frac{1}{5}$ or $y = 5$

when $y = -\frac{1}{5}$

$x = \frac{4 + \frac{3}{5}}{2} = \frac{23}{10} = 2.3$

when $y = 5$

$x = \frac{4 - 15}{2} = \frac{-11}{2} = -5.5$

$= (2.3, -0.2)$

or $(-5.5, 5)$

$x = 2.3$ or -5.5

$y = -0.2$ or 5

(5 marks)