Algebraic Proof

Exam Style Questions

1. Jane says that the product of any two prime numbers is always odd. Jane is wrong. Explain why.

Two is an even prime number and anything multiplied by two is even.

(1 mark)

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2. Jackson thinks that $n^2 + 5n + 1$ is always a prime number for all integer values of n. Is he correct?

Yes ☑ No ☐

Give a reason to support your answer.

$n^2 + 5n + 1$ cannot be factorised.

(2 marks)

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3. n is a positive integer. Aria says that expression $n^2 + 3n + 2$ can never be prime. Explain why Aria is correct.

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(2 marks)

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4. Susan says that the product of a square number and a cube number is always even. Give an example to prove that Susan is wrong.

$$\square \times \square = \square$$

(1 mark)

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5. A is a positive even integer. B is a negative integer. Complete the following table.

	Sometimes true	Always true	Never true.
AB is odd			
$A + B$ is positive.			
A^3B is negative.			

(2 marks)

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6. Match up the following descriptions to expressions:

Any two numbers	$n^2, \ m^2$
Two consecutive numbers	$2n + 1, \ 2n + 3$
Two consecutive even numbers	$n, \ \ n + 1$
Two consecutive odd numbers	$n, \ m$
Any two even numbers	$2n, \ 2m$
Any two odd numbers	$8n, \ 8m$
Two square numbers	$2n, \ 2n + 2$
Two multiples of 8	$2n + 1, \ 2m + 1$

(3 marks)

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7. Prove algebraically that the sum of any odd integer and any even integer is always odd.

odd + even

$= (2n+1) + (2m)$

$= 2(n+m)+1$

odd.

(2 marks)

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8. Prove algebraically that the sum of any two consecutive even integers is two more than a multiple of four.

$(2n) + (2n+2)$

$= 4n + 2$

↳ Two more than a multiple of 4.

(2 marks)

9. Prove algebraically that the product of any two consecutive even integers is always a multiple of four.

$2n(2n+2) = 4n^2 + 4n$

$= 4(n^2 + n)$

↳ multiple of 4.

(2 marks)

10. Prove that the product of any two consecutive odd integers is three more than a multiple of four.

$(2n+1)(2n+3)$

$= 4n^2 + 6n + 2n + 3$

$= 4n^2 + 8n + 3$

$= 4(n^2 + 2n) + 3$

↳ 3 more than a multiple of 4.

(2 marks)

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11. Prove that the sum of any two even integers is always even.

$2n + 2m = 2(n+m)$

↳ even.

(2 marks)

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12. Prove that the sum of any two consecutive numbers is odd.

$n + n + 1 = 2n + 1$

↳ odd

(2 marks)

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13. Prove that the sum of the squares of any two consecutive odd numbers is two more than a multiple of eight.

$(2n+1)^2 + (2n+3)^2$

$= (2n+1)(2n+1) + (2n+3)(2n+3)$

$= 4n^2 + 2n + 2n + 1 + 4n^2 + 6n + 6n + 9$

$= 8n^2 + 16n + 10$

$= 8(n^2 + 2n + 1) + 2$

↳ 2 more than a multiple of 8.

(3 marks)

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14. Prove the sum of the squares of two consecutive multiples of four is always a multiple of 16.

$(4n)^2 + (4n+4)^2$

$= 16n^2 + (4n+4)(4n+4)$

$= 16n^2 + 16n^2 + 16n + 16n + 16$

$= 32n^2 + 32n + 16$

$= 16(2n^2 + 2n + 1)$

↳ multiple of 16.

(3 marks)

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15. Prove that the difference of the squares of any two consecutive integers is the same as the sum of those two integers.

$(n+1)^2 - (n)^2$

$= (n+1)(n+1) - n^2$

$= n^2 + n + n + 1 - n^2$

$= 2n+1$

$= (n+1) + n$

(3 marks)

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16. Prove that the sum of any three consecutive even integers is always a multiple of six.

$(2n) + (2n+2) + (2n+4)$

$= 6n + 6$

$= 6(n+1)$

↳ multipre of 6.

(2 marks)

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17. Prove that the sum of $n(n + 3)$ and $3n + 9$ is always a square number for integer values of n.

$n(n+3) + 3n + 9$

$= n^2 + 3n + 3n + 9$

$= n^2 + 6n + 9$

$= (n+3)^2$

↳ square number.

(3 marks)

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18. Prove that $4(x + 2)^2 + 2(x - 1)^2$ is always a multiple of six for all integer values of x.

$4(x+2)(x+2) + 2(x-1)(x-1)$

$= 4(x^2+2x+2x+4) + 2(x^2-x-x+1)$

$= 4(x^2+4x+4) + 2(x^2-2x+1)$

$= 4x^2+16x+16+2x^2-4x+2$

$= 6x^2+12x+18$

$= 6(x^2+2x+3)$

↳ multiple of 6.

(3 marks)

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19. Prove that $(7x + 1)^2 - (3x + 1)^2$ is always a multiple of 8 for all integer values of x.

$(7x+1)(7x+1) - (3x+1)(3x+1)$

$= (49x^2+7x+7x+1) - (9x^2+3x+3x+1)$

$= (49x^2+14x+1) - (9x^2+6x+1)$

$= 40x^2+8x$

$= 8(5x^2+x)$

↳ multiple of 8.

(3 marks)

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20. Prove that $(5x + 1)^2 - (3x + 1)^2$ is always a multiple of 4 for all integer values of x.

$= (5x+1)(5x+1) - (3x+1)(3x+1)$

$= (25x^2+5x+5x+1) - (9x^2+3x+3x+1)$

$= (25x^2+10x+1) - (9x^2+6x+1)$

$= 16x^2+4x$

$= 4(4x^2+x)$

↳ multiple of 4.

(3 marks)

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21. Prove that the sum $\frac{1}{2}n(n + 2)$, $\frac{1}{2}n(n + 10)$ and $4n + 25$ is always a square number for all integer values of n.

$\frac{1}{2}n(n+2) + \frac{1}{2}n(n+10) + 4n+25$

$= \frac{1}{2}n^2+n + \frac{1}{2}n^2+5n + 4n+25$

$= n^2+10n+25$

$= (n+5)^2$

↳ square number.

(3 marks)

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22. Prove that the sum of the squares of any two consecutive even integers is always a multiple of 4.

$$(2n)^2 + (2n+2)^2 = 4n^2 + (2n+2)(2n+2)$$
$$= 4n^2 + 4n^2 + 4n + 4n + 4$$
$$= 8n^2 + 8n + 4$$
$$= 4(2n^2 + 2n + 1)$$

↳ multiple of 4.

(3 marks)

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23. Prove that $(n + 1)^2 + 3n(n + 2) + 10$ is always even for all integer values of n.

$$(n+1)(n+1) + 3n^2 + 6n + 10$$
$$= n^2 + 2n + 1 + 3n^2 + 6n + 10$$
$$= 4n^2 + 8n + 11$$
$$= 4\left(n^2 + 2n + \frac{11}{4}\right)$$
$$= 4\left[(n+1)^2 - 1 + \frac{11}{4}\right]$$
$$= 4\left[(n+1)^2 + \frac{7}{4}\right]$$
$$= \underbrace{4(n+1)^2}_{\text{positive}} + \underbrace{7}_{\text{positive}} > 0$$

(3 marks)

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24. Below are the first five terms of an arithmetic sequence

$$3,\ 7,\ 11,\ 15,\ 19$$

Prove that the difference of the squares of any two consecutive terms is always a multiple of 8.

nth term: $4n-1$

Two consecutive terms: $4n-1$, $4n+3$

$(4n+3)^2 - (4n-1)^2$

$= (4n+3)(4n+3) - (4n-1)(4n-1)$

$= (16n^2 + 24n + 9) - (16n^2 - 8n + 1)$

$= 32n + 8$

$= 8(4n+1)$

↳ multiple of 8.

(3 marks)

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25. x is an integer. Jasmine says that $x^2 - (x+3)(x+5)$ is always negative. Jasmine is wrong. Explain why.

$x^2 - (x+3)(x+5) = x^2 - (x^2 + 8x + 15) = -(8x+15)$

when $x = -2$, $-(8 \times -2 + 15) = 1$ which is positive.

Therefore Jasmine is wrong.

(2 marks)

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26. Prove that the sum of $\frac{1}{2}n(n+6)$, $\frac{1}{2}(n+2)(n+4)$ and $2(3n+16)$ is always a square number for all integer values of n.

$\frac{1}{2}n(n+6) + \frac{1}{2}(n+2)(n+4) + 2(3n+16)$

$= \frac{1}{2}n^2 + 3n + \frac{1}{2}(n^2 + 6n + 8) + 6n + 32$

$= \frac{1}{2}n^2 + 3n + \frac{1}{2}n^2 + 3n + 4 + 6n + 32$

$= n^2 + 12n + 36$

$= (n+6)^2$

$\hookrightarrow$ square number.

(3 marks)