Algebraic Proof

Exam Style Questions

1. Jane says that the product of any two prime numbers is always odd. Jane is wrong. Explain why.

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(1 mark)

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2. Jackson thinks that $n^2 + 5n + 1$ is always a prime number for all integer values of n. Is he correct?

Yes ☐ No ☐

Give a reason to support your answer.

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(2 marks)

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3. n is a positive integer. Aria says that expression $n^2 + 3n + 2$ can never be prime. Explain why Aria is correct.

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(2 marks)

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4. Susan says that the product of a square number and a cube number is always even. Give an example to prove that Susan is wrong.

$\square \times \square = \square$

(1 mark)

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5. A is a positive even integer. B is a negative integer. Complete the following table.

	Sometimes true	Always true	Never true.
AB is odd			
$A + B$ is positive.			
A^3B is negative.			

(2 marks)

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6. Match up the following descriptions to expressions:

Any two numbers	$n^2, \; m^2$
Two consecutive numbers	$2n + 1, \; 2n + 3$
Two consecutive even numbers	$n, \; n + 1$
Two consecutive odd numbers	$n, \; m$
Any two even numbers	$2n, \; 2m$
Any two odd numbers	$8n, \; 8m$
Two square numbers	$2n, \; 2n + 2$
Two multiples of 8	$2n + 1, \; 2m + 1$

(3 marks)

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7. Prove algebraically that the sum of any odd integer and any even integer is always odd.

(2 marks)

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8. Prove algebraically that the sum of any two consecutive even integers is two more than a multiple of four.

(2 marks)

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9. Prove algebraically that the product of any two consecutive even integers is always a multiple of four.

(2 marks)

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10. Prove that the product of any two consecutive odd integers is three more than a multiple of four.

(2 marks)

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11. Prove that the sum of any two even integers is always even.

(2 marks)

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12. Prove that the sum of any two consecutive numbers is odd.

(2 marks)

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13. Prove that the sum of the squares of any two consecutive odd numbers is two more than a multiple of eight.

(3 marks)

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14. Prove the sum of the squares of two consecutive multiples of four is always a multiple of 16.

(3 marks)

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15. Prove that the difference of the squares of any two consecutive integers is the same as the sum of those two integers.

(3 marks)

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16. Prove that the sum of any three consecutive even integers is always a multiple of six.

(2 marks)

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17. Prove that the sum of $n(n + 3)$ and $3n + 9$ is always a square number for integer values of n.

(3 marks)

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18. Prove that $4(x + 2)^2 + 2(x - 1)^2$ is always a multiple of six for all integer values of x.

(3 marks)

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19. Prove that $(7x + 1)^2 - (3x + 1)^2$ is always a multiple of 8 for all integer values of x.

(3 marks)

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20. Prove that $(5x + 1)^2 - (3x + 1)^2$ is always a multiple of 4 for all integer values of x.

(3 marks)

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21. Prove that the sum $\frac{1}{2}n(n + 2)$, $\frac{1}{2}n(n + 10)$ and $4n + 25$ is always a square number for all integer values of n.

(3 marks)

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22. Prove that the sum of the squares of any two consecutive even integers is always a multiple of 4.

(3 marks)

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23. Prove that $(n + 1)^2 + 3n(n + 2) + 10$ is always even for all integer values of n.

(3 marks)

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24. Below are the first five terms of an arithmetic sequence

$$3,\ 7,\ 11,\ 15,\ 19$$

Prove that the difference of the squares of any two consecutive terms is always a multiple of 8.

(3 marks)

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25. x is an integer. Jasmine says that $x^2 - (x + 3)(x + 5)$ is always negative. Jasmine is wrong. Explain why.

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(2 marks)

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26. Prove that the sum of $\frac{1}{2}n(n + 6)$, $\frac{1}{2}(n + 2)(n + 4)$ and $2(3n + 16)$ is always a square number for all integer values of n.

(3 marks)