

## Completing the Square

### Exam Style Questions

1. Complete the square of  $x^2 + 8x + 4$

$$\begin{aligned} x^2 + 8x + 4 &= (x+4)^2 - 4^2 + 4 \\ &= (x+4)^2 - 16 + 4 \\ &= (x+4)^2 - 12 \end{aligned}$$

.....  $(x+4)^2 - 12$  ..... (2 marks)

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2. Complete the square of  $x^2 + 12x + 6$

$$\begin{aligned} x^2 + 12x + 6 &= (x+6)^2 - 6^2 + 6 \\ &= (x+6)^2 - 36 + 6 \\ &= (x+6)^2 - 30 \end{aligned}$$

.....  $(x+6)^2 - 30$  ..... (2 marks)

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3. Complete the square of  $x^2 + 20x - 4$

$$\begin{aligned} &= (x+10)^2 - 10^2 - 4 \\ &= (x+10)^2 - 100 - 4 \\ &= (x+10)^2 - 104 \end{aligned}$$

.....  $(x+10)^2 - 104$  ..... (2 marks)

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4. (a) Write  $x^2 + 12x + 6$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are integers.

$$= (x+6)^2 - 6^2 + 6$$

$$= (x+6)^2 - 36 + 6$$

$$= (x+6)^2 - 30$$

$$\dots (x+6)^2 - 30 \dots$$

- (b) Hence, write down the coordinates of the turning point of the graph with equation

$$y = x^2 + 12x + 6$$

$$\dots (-6, -30) \dots \quad (3 \text{ marks})$$


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5. (a) Write  $x^2 + 9x - 1$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are constants to be determined.

$$= (x+4.5)^2 - 4.5^2 - 1$$

$$= (x+4.5)^2 - 20.25 - 1$$

$$= (x+4.5)^2 - 21.25$$

$$\dots (x+4.5)^2 - 21.25 \dots$$

- (b) Hence, write down the coordinates of the turning point of the graph with equation

$$y = x^2 + 9x - 1$$

$$\dots (-4.5, -21.25) \dots \quad (3 \text{ marks})$$


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6. (a) Write  $x^2 - 20x + 3$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are constants to be determined.

$$= (x-10)^2 - 10^2 + 3$$

$$= (x-10)^2 - 100 + 3$$

$$= (x-10)^2 - 97$$

$$\dots\dots\dots (x-10)^2 - 97$$

- (b) Hence, write down the coordinates of the turning point of the graph with equation  $y = x^2 - 20x + 3$

$$\dots\dots\dots (10, -97) \dots\dots\dots (3 \text{ marks})$$

7. By completing the square, find the coordinates of the turning point of the graph with equation  $y = x^2 - x - 5$ .

$$y = \left(x - \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 5$$

$$= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 5$$

$$= \left(x - \frac{1}{2}\right)^2 - \frac{21}{4} \Rightarrow \text{TP} = \left(\frac{1}{2}, -\frac{21}{4}\right)$$

$$= (0.5, -5.25)$$

$$\dots\dots\dots (0.5, -5.25) \dots\dots\dots (3 \text{ marks})$$

8. By completing the square, find the coordinates of the turning point of the graph with equation  $y = 5x^2 + 10x + 20$ .

$$\begin{aligned}
 y &= 5[x^2 + 2x + 4] \\
 &= 5[(x+1)^2 - 1^2 + 4] \\
 &= 5[(x+1)^2 + 3] \\
 &= 5(x+1)^2 + 15 \Rightarrow \text{TP} = (-1, 15)
 \end{aligned}$$

.....  $(-1, 15)$  ..... (4 marks)

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9. By completing the square, find the coordinates of the turning point of the graph with equation  $y = 3x^2 + 12x + 18$ .

$$\begin{aligned}
 y &= 3[x^2 + 4x + 6] \\
 &= 3[(x+2)^2 - 2^2 + 6] \\
 &= 3[(x+2)^2 - 4 + 6] \\
 &= 3[(x+2)^2 + 2] \\
 &= 3(x+2)^2 + 6
 \end{aligned}$$

.....  $(-2, 6)$  ..... (4 marks)

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10. By completing the square, find the coordinates of the turning point of the graph with equation  $y = 2x^2 + 8x + 9$ .

$$\begin{aligned}
 y &= 2[x^2 + 4x + 4.5] \\
 &= 2[(x+2)^2 - 2^2 + 4.5] \\
 &= 2[(x+2)^2 - 4 + 4.5] \\
 &= 2[(x+2)^2 + 0.5] \\
 &= 2(x+2)^2 + 1
 \end{aligned}$$

.....  $(-2, 1)$  ..... (4 marks)

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11. By completing the square, find the coordinates of the turning point of the graph with equation  $y = 3x^2 + 7x - 3$ .

$$y = 3 \left[ x^2 + \frac{7}{3}x - 1 \right]$$

$$= 3 \left[ \left( x + \frac{7}{6} \right)^2 - \left( \frac{7}{6} \right)^2 - 1 \right]$$

$$= 3 \left[ \left( x + \frac{7}{6} \right)^2 - \frac{49}{36} - 1 \right]$$

$$= 3 \left[ \left( x + \frac{7}{6} \right)^2 - \frac{85}{36} \right] = 3 \left( x + \frac{7}{6} \right)^2 - \frac{85}{12} \quad \dots \left( -\frac{7}{6}, -\frac{85}{12} \right) \dots \text{ (4 marks)}$$

12. By completing the square, solve the equation  $x^2 + 6x + 2 = 0$  leaving your solutions in the form  $a \pm b\sqrt{c}$  where  $a, b$  and  $c$  are integers.

$$(x+3)^2 - 3^2 + 2 = 0$$

$$(x+3)^2 - 9 + 2 = 0$$

$$(x+3)^2 - 7 = 0$$

$$(x+3)^2 = 7$$

$$x+3 = \pm\sqrt{7}$$

$$x = -3 \pm \sqrt{7}$$

$$\dots x = -3 \pm \sqrt{7} \dots \text{ (4 marks)}$$

13. By completing the square, solve the equation  $x^2 + 7x + 11 = 0$  leaving your solutions in surd form.

$$(x+3.5)^2 - 3.5^2 + 11 = 0$$

$$(x+3.5)^2 - 12.25 + 11 = 0$$

$$(x+3.5)^2 - 1.25 = 0$$

$$(x+3.5)^2 = 1.25$$

$$x+3.5 = \pm\sqrt{1.25}$$

$$x = -3.5 \pm \sqrt{1.25}$$

$$= -\frac{7}{2} \pm \sqrt{\frac{5}{4}}$$

$$= \frac{-7 \pm \sqrt{5}}{2}$$

$$\text{or } x = \frac{-7 + \sqrt{5}}{2}$$

$$x = \frac{-7 - \sqrt{5}}{2}$$

$$\dots \text{ (4 marks)}$$

14. By completing the square, solve the equation  $2x^2 + 8x - 2 = 0$  leaving your solutions in the form  $a \pm b\sqrt{c}$  where  $a, b$  and  $c$  are integers.

$$2[x^2 + 4x - 1] = 0$$

$$x^2 + 4x - 1 = 0$$

$$(x+2)^2 - 2^2 - 1 = 0$$

$$(x+2)^2 - 5 = 0$$

$$(x+2)^2 = 5$$

$$x+2 = \pm\sqrt{5}$$

$$x = -2 \pm \sqrt{5}$$

$$\dots\dots\dots x = -2 \pm \sqrt{5} \quad (5 \text{ marks})$$

15. By completing the square, solve the equation  $-x^2 - 7x + 2 = 0$  leaving your solutions in surd form.

$$-x^2 - 7x + 2 = 0$$

$$\Rightarrow x^2 + 7x - 2 = 0$$

$$\Rightarrow (x + 3.5)^2 - 3.5^2 - 2 = 0$$

$$\Rightarrow (x + 3.5)^2 - 12.25 - 2 = 0$$

$$(x + 3.5)^2 - 14.25 = 0$$

$$(x + 3.5)^2 = 14.25$$

$$x + 3.5 = \pm\sqrt{14.25}$$

$$x = -3.5 \pm \sqrt{14.25}$$

$$= -\frac{7}{2} \pm \sqrt{\frac{57}{4}}$$

$$= \frac{-7 \pm \sqrt{57}}{2}$$

$$\dots\dots\dots x = \frac{-7 \pm \sqrt{57}}{2} \quad (5 \text{ marks})$$

16. By completing the square, solve the equation  $ax^2 + bx + c = 0$ , leaving your answer in terms of  $a$ ,  $b$  and  $c$ .

$$a \left[ x^2 + \frac{b}{a}x + \frac{c}{a} \right] = 0$$

$$\Rightarrow x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\Rightarrow \left( x + \frac{b}{2a} \right)^2 - \left( \frac{b}{2a} \right)^2 + \frac{c}{a} = 0$$

$$\Rightarrow \left( x + \frac{b}{2a} \right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\Rightarrow \left( x + \frac{b}{2a} \right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\Rightarrow \left( x + \frac{b}{2a} \right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$\Rightarrow x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\Rightarrow x = \frac{-b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \dots \dots \dots (6 \text{ marks})$$